Let φ be an analytic self-map of the open unit disc $\mathbb{D}$ and u be a bounded complex - valued analytic function defined on $\mathbb{D}$. The linear map uC_φ on $H(\mathbb{D})$ defined by

$$(uC_\varphi)(f)(z) = u(z)(f \circ \varphi)(z), \forall f \in H(\mathbb{D}), \forall z \in \mathbb{D},$$

is called the **weighted composition operator** with symbol φ and weight u. For $\alpha > -1$, the weighted Bergman Space A^2_α consists of all analytic functions in $L^2(\mathbb{D}, dA_\alpha)$.

Our operator of interest is the difference of two weighted composition operators $uC_\varphi - vC_\psi$ acting between weighted Bergman spaces. We are particularly interested in characterizing the **Hilbert-Schmidtness** of that operator acting on A^2_α. In general, when u and v are both **arbitrary** bounded analytic functions, the problem seems to be **challenging**. The special case of $u = 1$ and $v = 1$ (the unweighted case) has been solved by Choe, Hosokawa and Koo in 2010. We will first discuss an alternative proof of their result. Then we will discuss various different cases where u and v are of some specific forms. A systematic development of proof techniques will be highlighted. At the end of the presentation, we will discover the "best" result in this direction so far.