A circle packing is an arrangement of tangent circles on a surface. These tangent circles approximate the infinitesimal circles preserved under holomorphic functions, so maps between packings provide a model for discrete conformal geometry. We will look at some recent progress in inversive distance packings, in which the circles are no longer tangent but have prescribed inversive distances. These recent results have shed new light on outstanding existence and uniqueness questions by bringing in new ideas from rigidity theory.