Error Correcting Codes and Finite Geometries

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For $n \geq k$, an $(n,k,d)_q$-*code* C is a collection of q^k n-tuples *(codewords)*) over an alphabet $\mathcal{A}$ of size q such that the minimum (Hamming) distance between any two codewords of C is d. In the special case that $\mathcal{A} = GF(q)$ and C is a vector space of dimension k over $GF(q)$, C is a *linear* $(n,k,d)_q$-code. For an (n,k,d)-code over an alphabet $\mathcal{A}$ the Singleton bound:$|C| \leq |\mathcal{A}|^{n-d+1}$ gives $d \leq n-k+1$. The *Singleton defect of* C, $S(C)$, is defined by $S(C) = n-k+1-d$.
A fundamental question is that of the maximum length n for codes of fixed parameters k, q and $S(C)$. Here, after an introduction to finite projective geometries and error correcting codes we shall discuss this question initially for (not necessarily linear) codes with $S(C) = 0$, and then broadening discussions. Several open problems will be presented.