Primitivity of non-noetherian group algebras

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A ring R is (right) primitive provided it has a faithful irreducible (right) R-module. For the group algebra KG of a group $G \neq 1$ over a field K, if KG is primitive then G is a non-abelian infinite group. Almost all known infinite groups except for polycyclic by finite groups belong to the class of non-noetherian groups. A group of the class of finitely generated non-noetherian groups has often non-abelian free subgroups; for instance, a free group, a locally free group, a free product, an amalgamated free product, an HNN-extension, a Fuchsian group, a one relator group, etc.

In this talk, we focus on a local property which is often satisfied by groups with non-abelian free subgroups:

($*$) For each subset M of G consisting of finite number of elements not equal to 1, there exist three distinct elements a, b, c in G such that whenever $x_i \in \{a, b, c\}$ and $(x_1^{-1}g_1x_1)\cdots(x_m^{-1}g_mx_m) = 1$ for some $g_i \in M$, $x_i = x_{i+1}$ holds for some i.

We can see that if G is a countably infinite group and satisfies ($*$), then KG is primitive for any field K. More generally, we get the following theorem:

Theorem Let G be a non-trivial group which has a free subgroup whose cardinality is the same as that of G. Suppose that G satisfies the condition ($*$). If R is a domain with $|R| \leq |G|$, then the group ring RG of G over R is primitive.

In particular, the group algebra KG is primitive for any field K.

In order to prove Theorem above, we define an SR-graph and an SR-cycle and use a graph-theoretic method.