Calculations with Ideals in Quadratic Domains

Abstract

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The development of ideals in the 19th century was motivated by the desire to restore a form of unique prime factorization to integral domains of algebraic integers, in which it is often possible to give examples of essentially distinct factorizations of integers into irreducible elements. Kummer first obtained unique factorization using what he called *ideal numbers*, presumably existing in some larger set of elements. Dedekind later refined Kummer's concept into the precise definition of ideal numbers as certain types of subsets of integral domains, which he called *ideals*, as one currently encounters in ring theory. However, it is likely that Dedekind still regarded these ideals as "numbers," in some sense, intended for specific computational applications with algebraic integers.

In this talk, I will describe my current research emphasizing this numerical interpretation of ideals in the special case of *quadratic domains*, that is, integral domains of algebraic integers in fields of degree two over the rational numbers. I will introduce a notation for ideals intended to make calculations more practical, and describe its applications to classification of prime ideals, prime factorization of ideals, multiplication and other operations on ideals, and equivalence of ideals and reduction of ideals into equivalence class representatives. The talk will also touch on applications of these concepts to particular arithmetic problems, such as representation of integers by quadratic forms, and description of solutions of Pell's equation.