Transformation Semigroups: Structure, Automorphisms, and Graphs

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A *semigroup* is a pair $(S, *)$, where S is a set and $*$ is an associative operation on S. The beginning of semigroup theory can be traced to an article published in 1928 by a Russian mathematician A. Suschkewitsch. If a semigroup S has the identity, then it is called a *monoid*. A *transformation semigroup* is a semigroup whose elements are transformations on a set X (functions with domain and image included in X) and whose operation is the composition of functions. In group theory, Cayley's Theorem states that every group is isomorphic to a group of permutations on some set X. We have an analogous result in semigroup theory: every semigroup is isomorphic to a semigroup of transformations on some set X.

I will focus on three themes that have been prominent in my study of transformation semigroups. One theme is the structure of a semigroup S in terms of Green's relations, which are equivalences on S defined using principal ideals of S. Another one is automorphisms of a semigroup S. Here one wants to describe the automorphisms of a given semigroup and determine the group of automorphisms. The third theme concerns relations between semigroups and graphs. There are various graphs that can be associated with a given semigroup. One can study properties of these graphs or obtain results about the semigroup itself using an associated graph. Given a transformation semigroup S, one can also represent elements of S as directed graphs and express properties of S in terms of these digraphs.

I will describe problems that arise in the study of semigroups along these lines, present some specific results that I obtained in the last years, and discuss some open questions.