PROPERTIES OF RINGS AND OF RING EXTENSIONS THAT ARE INVARIANT UNDER GROUP ACTION

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ABSTRACT. We expand the work in invariant theory inspired by Hilbert's Fourteenth Problem. Given a commutative ring with identity R and a subgroup G of the automorphism group of R, the *fixed ring* is $R^G := \{r \in R \mid \sigma(r) = r \text{ for all } \sigma \in G\}$. That is, R^G is the collection of elements of R that are fixed by all automorphisms in G.

We determine properties of ring extensions that are invariant under certain group action. Given an extension of commutative rings with identity $R \subset T$ and a subgroup G of the automorphism group of T, we have $R^G \subseteq T^G$, where these fixed rings are defined as above. We often assume that R is G-invariant, i.e., $\sigma(R) \subseteq R$ for all $\sigma \in G$. We call properties of the ring extension $R \subset T$ inherited by $R^G \subseteq T^G$ *invariant*. In particular, we consider minimal ring extensions, flat epimorphic extensions, extensions satisfying the finitely many intermediate algebras property or the finite chain property, and related extensions.